Thermodynamic Performance Evaluation of a Hydroduct Using a Thermite Fuel

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The interest in underwater propulsion schemes has directed attention to a number of devices based on the jet propulsion principle. This paper examines the thermodynamics of a system that bears similarity to the ramjet. The limitations of the system, however, are serious and cycle performance calculations suggest a very restricted range of operation. Both depth sensitivity and cavitation effects couple to limit effective operation. Sample optimal calculations are presented assuming isentropic behavior of all phases of the cycle.

I. Introduction

PROPULSION systems that can drive underwater vehicles at speeds greater than can be attained by conventional propeller systems have become attractive in this era of oceanographic development. Unfortunately, the technology of unconventional underwater propulsion has not been as advanced as its air counterpart. It is hoped that underwater propulsion systems will witness rapid progress in the future.

Marine engineers have noted that there exists some analogy between hydrodynamic and aerodynamic propulsion systems and have suggested that some principles embodied in the turbofan, turbojet, ramjet, and rocket propulsion systems be utilized for underwater application.[1] At low speeds, the classical incompressible fluid theory is applicable to both media. At higher speeds, however, the onset of cavitation in water renders any further analogy unrealistic. Cavitation is peculiar to the flow of liquids and occurs whenever local pressure at any point within the flow is reduced to the order of the vapor pressure of the liquid. Under these conditions, vaporization then takes place and gives rise to a two-phase mixture. Thus, one may liken the onset of cavitation to the aerodynamic sonic barrier. As yet there has not been a satisfactory analysis of this process. The effect of cavitation is to increase the drag which in turn reduces the effective thrust of any submersible. Therefore, in the absence of suitable design criteria to delay the onset of vaporization, high-speed underwater devices must be limited to relatively low speeds and to a limited number of hydrodynamic shapes.

A recent propulsion scheme that appears promising for high-speed underwater application is the hydroduct, a device that can be likened to a ramjet.[2,3] Water is introduced at ram pressure and then sprayed over the water reactive fuel where it is vaporized. The high-pressure heated water and steam mixture is then expanded through a critical nozzle and is exhausted at the rear of the device as a high-pressure jet that propels the vehicle forward. The high heat released by the fuel produces a high thrust level which can be incorporated into a compact and minimum drag vehicle. Marine applications for such a propulsion scheme are numerous. The simplicity of the system lies in the fact that it requires no moving parts; however, it is the purpose of this paper to dwell primarily upon a general thermodynamic cycle analysis of the ramjet principle as it applies to an underwater propulsion system and, in addition, to consider the potentialities and possible limitations of this cycle.

II. General Considerations

The underwater ramjet is a propulsion system which differs from its aerodynamic counterpart in that the working fluid undergoes a change in phase and a significant increase in volume during the heating portion of the cycle. Two such designs are shown in Figs. 1a and 1b, which differ only in the method in which water is admitted to the combustion chamber. A monopropellent heat source provides the energy for propulsion. Unfortunately, the high specific impulse normally associated with airborne ramjets and rockets cannot be achieved in the underwater application and certain advantages inherent in the ramjet can not practically be embodied in the device. One major reason rests clearly in the nature of the working substance. Most of the energy release is only used to raise a sensible heat of water to its saturation condition while the remainder produces a phase change. The useful work is done by the steam vapor, whereas the hot saturated water which absorbs a major portion of available heat does not contribute to propulsion. It can be said that the main advantage of the underwater ramjet may be found in its higher efficiency over conventional propeller driven devices. However, the hydroduct must be launched at an initially high speed in order to develop sufficient ram pressure to sustain its operation. One finds that, whereas thrust is proportional to ram pressure, frictional drag and ram pressure are both proportional to the square of the forward velocity. For a range of velocities, then, the net thrust, or total thrust minus drag, is practically a constant, as shown in Fig. 2. The drag coefficient is obtained by approximating air flow over similar shapes and correcting for viscosity. However, as cavitation conditions are approached, the net thrust is sharply reduced to zero. Therefore, for a hydroduct launched at near cavitating velocities, the drag effects will determine its final velocities and thrust conditions. If the idealized thrust conditions are not realized, an all actual drag losses

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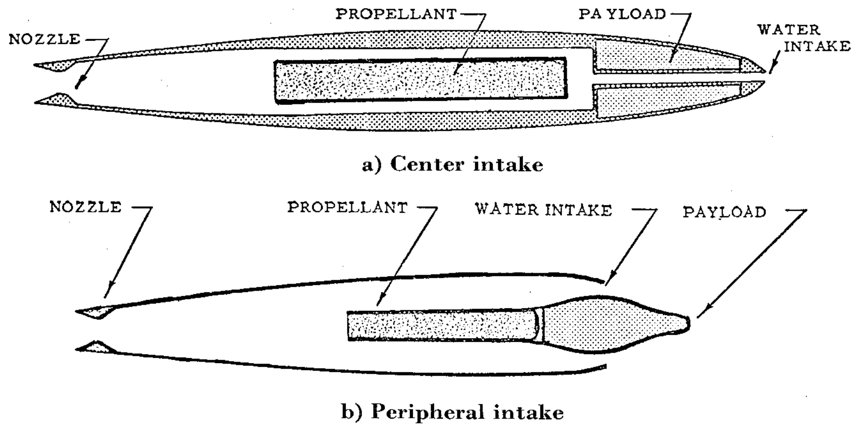

Fig. 1 Hydroduct.

are greater than anticipated, then the drag will exceed thrust for all forward velocities and the hydroduct will always decelerate. Results of the sample design calculations indicate that the thrust is dependent upon the following factors: 1) water rate into the hydroduct combustion chamber, 2) the rate of heat release from the propellant, 3) the initial launching velocity of the hydroduct, and 4) the depths of operation.

III. Thermodynamic Design Calculations

One may consider the hydroduct as a thermodynamic cycle in which water is introduced into the combustion chamber where it is vaporized at its stagnation pressure and exhausted as a high-velocity, low-quality steam jet through a critical nozzle behind the vehicle. If it is assumed that the propellant acts as a heat source, supplying energy to vaporize water, the complete cycle may then be mapped on a Mollier steam chart, as shown in Fig. 3. The initial conditions are total static pressure equal to the sum of atmospheric and hydrostatic pressure and stagnation conditions are dependent primarily on the forward velocity of the vehicle. Water is heated adiabatically and at constant pressure in the combustion chamber (2–3). It is further assumed that the ram intake passage is designed to minimize pressure drop. Under these restrictions, the available heat from the fuel is consumed by increasing the sensible heat of water and partially vaporizing the water. The state of the mixture of water and steam through the nozzle is determined by assuming an ideal isentropic process to final ambient conditions (3–4). The complete thermodynamic cycle is shown in Fig. 3 and a sample calculation for a given set of design parameters is given in Appendix A.

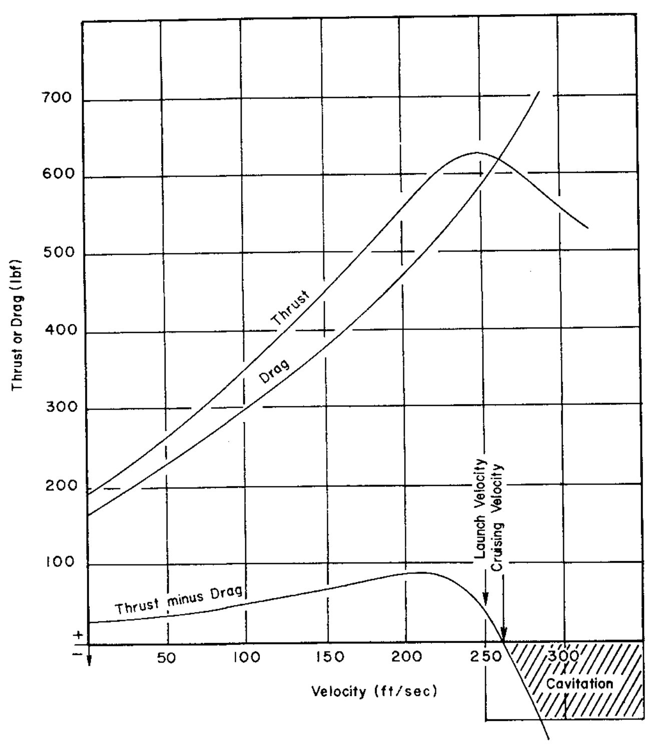

Fig. 2 Thrust and drag vs velocity.

IV. Water Intake Design

The water intake passage is critical in its effect on over-all performance. The proper design of the passage, namely the hydraulic radius, the length of the intake passage, and the location of the intake port, is necessary to obtain a minimum pressure drop. Optimal conditions for the mass flow rate of water into the combustion chamber are necessary since it is clear that too high a water rate will quench the reaction, with the result that steam pressure is reduced, which in turn produces a reduction in thrust. Too small a water rate, on the other hand, will result in superheating the steam to a pressure high enough to risk choking at the nozzle. An increase in chamber pressure directly reduces the mass of water entering the duct, with a corresponding decrease in thrust. Summarizing, then, too large an intake passage does not produce the degree of vaporization required for high thrust, whereas too small an intake passage does not provide sufficient mass flow necessary to maintain adequate thrust. For each forward velocity, one must match the mass flow rate of water intake area in order to operate at the maximum velocity.

The length-to-diameter ratio of the intake passage and the mass flow rate of water determine the pressure drop between the intake port and the combustion chamber. The requirements for high-speed and shallow running depth operation defines the slenderness ratio and hydrodynamic shape profile of the hydroduct that must be used in order to avoid the

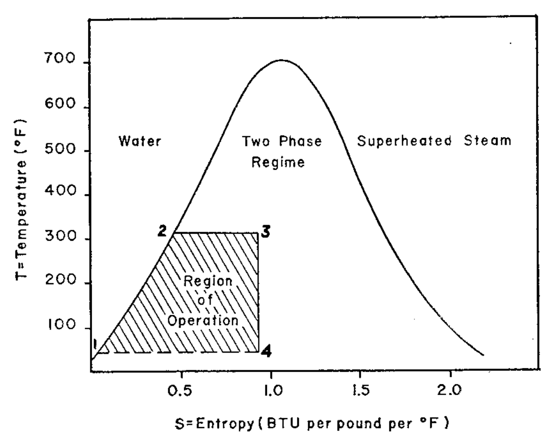

Fig. 3 Hydroduct thermodynamic cycle (equilibrium expansion process).

effects of cavitation. The payload and combustion chamber are then placed in a tandem arrangement, which requires that the water intake passage be located along the axis of the body and that the opening be located at the stagnation point. Such a configuration is shown in Fig. 1a. The length-to-diameter ratio of the intake passage must be large to maximize the payload space. In typical cases, for large length-to-diameter ratio and high water flow rates, as much as a 100-psi pressure drop can be expected in the passage. A large pressure drop then may affect the pressure sensitive reaction in the combustion chamber with result that it will not produce a sufficient rate of heat release to maintain an adequate thrust. Therefore, the length-to-diameter ratio of the intake passage must be optimized insofar as it is practical with due regard to its influence on the combustion chamber operating pressure.

The importance of the water flow rate can be illustrated by considering that at a water mass flow rate of 8 lb/sec the thrust is found to be 377 lbf, as shown in Appendix A. With an increase in water rate to 14 lb/sec only 197 lb of thrust is produced. This result is based upon the assumption of a "two-phase" expansion process through the nozzle. A comparison of this method with one based on the so-called "equilibrium" expansion process in the nozzle is illustrated in Appendix A. Both of these calculations are described in detail.

A further example of the influence of water mass flow can be obtained by considering a flow rate of 3.5 lb/sec such that the propellant could completely vaporize the intake water, bringing it to a superheated state before expansion. Under these conditions, the design point could probably not be achieved because of the excessively high pressures that are produced in the combustion chamber, result in choked flow in the nozzle throat.

It may be concluded that operation within a two-phase region of the Mollier chart, shown in Fig. 3, is required to develop sufficient thrust. As the water intake area is reduced the thrust increases. Within the superheated regions, the calculations indicate that the thrust is less sensitive and should continue to increase, but at a reduced rate. A plot of thrust vs water intake rate is shown in Fig. 4. The parameter of forward velocity is shown intersecting the two arbitrary intake area curves. As the area is increased, there is a corresponding decrease in thrust for each velocity. It appears that the optimum performance is achieved where the velocity curve is a maximum. The operational limits are quite narrow. Forward velocity increases with an increase in the sensitivity of the vehicle to intake area. However, the high temperatures associated with this regime make for the larger heat losses that may also cause a reduction in thrust.

V. Nozzle Design

It is clear that the nozzle is an important factor in the regulation of flow of the working fluid. The working fluid is presumed to be a three-phase mixture expansion process of wet steam and solid reaction products expanding through a nozzle; although an exact understanding of the expansion process in such a three-phase mixture is not available, some approximate models can be used.

One such idealized model assumes that the water droplets and steam vapor form a homogeneous mixture and there is no slip between liquid and vapor in each section of the nozzle.

The exhaust jet velocity is computed directly from the adiabatic energy relation,

$$V_f^2 = V_h^2 + 2Jg_c\Delta h \tag{1}$$

The thrust is computed from the following equation:

$$T = m/g_c[(2Jg_c\Delta h - V_h^2) - V_h]^{1/2} \tag{2}$$

Where m = mass rate of flow, V_f = velocity of jet, V_h = velocity of hydroduct, Δh = enthalpy change of mixture across nozzle, and g_c = consistency factor. This model overestimates the thrust available by neglecting the thrust available from the solid particles and liquid droplets.

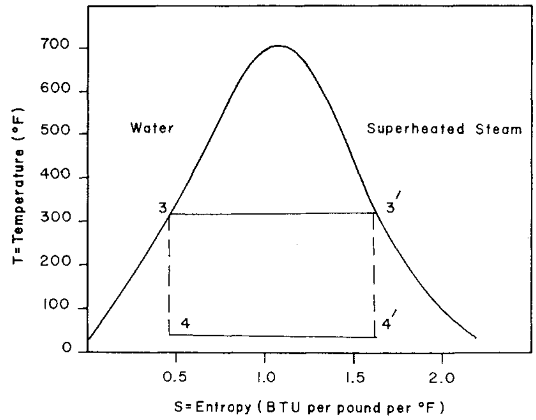

Fig. 4 Hydroduct thermodynamic cycle (two-phase expansion process).

Using such an equilibrium expansion process, it can be shown that the nozzle is extremely sensitive to local ambient conditions. The critical expansion process is dependent upon the *ratio* of the combustion chamber pressure to the back pressure at the nozzle exit. High stagnation pressures and low back pressures are most desirable for full expansion of the exhaust jet.[4] Therefore, the maximum operating condition may be achieved at a high forward velocity and shallow depths. Cavitation is an upper limiting condition on the attainable velocity. A second consideration also affects the hydroduct performance. Yellott[5] has observed that during the expansion of a wet steam mixture through a supersonic nozzle, there exists a supersaturation limit of expansion beyond which the fluid no longer behaves as an ideal homogeneous fluid, but depends upon the initial drop size of the moisture in the wet steam. If large liquid drops are present (10^{-5} in. in diameter) a noticeable increase in pressure is observed. Below a drop size less than 10^{-5} in. in diameter the effect is minimized. For a smooth transition to the mist state new droplet nucleii, approximately equal the original ones, are formed. Kennan[6] has calculated this pressure rise for several conditions of drop sizes.

The expansion of a condensible vapor through supersonic nozzle is accompanied by the formation of a condensation shock. The liquid volume fraction after condensation is a function of the upstream pressure conditions. For high degrees of superheat, the liquid fraction can be minimized and the expansion process can be successfully treated as an ideal isentropic process. As the liquid fraction increases, with decreasing degrees of superheat, the condensation shock produces an increase in pressure.[7] Local shear stress that can normally be neglected with gas expansion becomes significant with increasing liquid volume fraction. The net effect of pressure and frictional losses is to reduce the effective thrust of the vehicle.

An approximate method of computing the various states of wet steam during the expansion process was proposed by Church.[8] Since steam in a wet state will not behave as a uniform equilibrium mixture, water droplets, because of their greater mass, will not travel as rapidly as the vapor. As the pressure continues to drop further during the stages of vapor acceleration through the nozzle, the water droplets will ultimately settle out along the walls. Church[8] proposed a process whereby the saturated liquid and the saturated vapor can be considered separately; both expand isentropically to final ambient pressure conditions and the resultant enthalpy is used to compute the velocities of the liquid droplets and

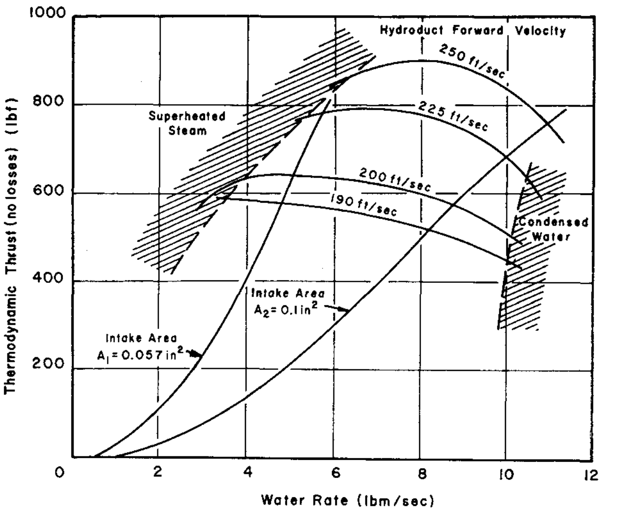

Fig. 5 Two phase expansion of steam and solid particles.

vapor. This method is illustrated in Appendix A. We have also extended this procedure to account for the solid particles present in the fluid steam as well. It can be shown that jet velocity computed in this manner will always be smaller than the final velocity estimated from the complete equilibrium method.

The jet velocity exhausted from the hydroduct furnishes the required thrust, but it is dependent upon the effective back pressure in the chamber. For a given forward vehicle velocity, i.e., at a given combustion chamber pressure, a high back pressure causes the steam to separate from the nozzle walls and decrease the jet velocity. Thus, the hydroduct will exhibit a reduced thrust with an increase in depth. However, there is a gain in thrust at increased depths. Although the combustion chamber operates at a higher pressure level, the propellant burns faster, improving the steam quality. The losses encountered at a greater back pressure at the nozzle exit are shown to be greater than the gain due to any improvement in steam quality. The net effect of increased depth is therefore to reduce performance. The effect of depth on thrust is shown in Fig. 6. Several design schemes purport to reduce depth sensitivity. Each entails some loss in thrust performance. This loss, although small, is sufficient to eliminate any positive thrust of the hydroduct and therefore subjects the vehicle to dynamic instability along its trajectory.

In the following example, one method is described in which sensitivity to depth may be achieved only at the expense of a reduction in performance requirements. If the hydroduct is capable of achieving, for example, 580 lb of thrust at 50 ft, assuming ideal nozzle conditions, only 490 lb of thrust can be obtained at a depth of 500 ft. Losses incurred by an overexpansion of the jet at the increased pressure level are the major reason for this reduction in thrust. If a nozzle with a smaller exit area is used so that optimum expansion occurs at the same depth, thrust will be approximately 490 lb for both the shallow water operation as well as the deep water operation. In this simplified illustration the depth sensitivity can be achieved only by some compromise in performance in shallow water. In actual operation, the jet may recover some of its initial thrust even during operation; at shallow depth this is due to an underexpansion of the jet and due to the fact that mixing does not occur efficiently between the water-air interface. It acts more as an extension of the nozzle wall. For a given depth, an increase in the stagnation pressure, which results from a higher forward velocity, tends to increase the burning rate of the propellant, which results in a higher net thrust. For a fixed propellant grain, the vehicle will operate at a speed slightly higher than design. The increase in speed may prove to be more efficient at greater depths, since the increase in pressure delays the onset of cavitation.

These effects on range of the hydroduct require separate calculations for each set of hydroduct design parameters.

VI. Compatability Conditions for Hydroduct Design

In the preceding discussion, the calculations were made neglecting the effect of pressure changes across the intake passage and the subsequent effect on the mass flow rate. The pressure drop through the intake determines the mass flow of water entering the combustion chamber and also dictates the operating conditions in the combustion chamber. Changes in chamber pressure specify the heat release and steam quality. The pressure drop also controls the stagnation pressure necessary to force the steam through the nozzle throat. The nozzle throat area is critical to insure supersonic flow to achieve a realistic design condition. A possible design may be checked against ideal thrust calculations, which must be at least 10% greater than the drag forces for a given forward velocity. If the net thrust is negative, then the design velocity cannot be attained and another set of design criteria must be selected to satisfy successful operating conditions.

VII. Conclusions

1) An underwater hydroduct design for 300 sec of specific impulse when compared with an underwater rocket at the same velocity and the same range appears to be the more successful approach, although both are severely limited in performance.

2) Only a small fraction of the total thermal energy released by the hydroductor propellant is converted into useful work. The greatest portion is used in heating the liquid to its saturation conditions.

3) A thermodynamic analysis based on an isentropic analysis has shown that the hydroduct does not develop sufficient thrust to overcome drag effects. Losses incurred as a result of an expansion of a two-phase mixture of steam through the nozzle have been neglected in this analysis. Momentum losses introduced by particulate solids produced during reaction, the extent of reaction between the water and the propellant, and functional losses in the jet can be expected to further reduce the net thrust.

For probable design conditions where velocity is assumed to be 200 fps, an ideal homogeneous isentropic expansion

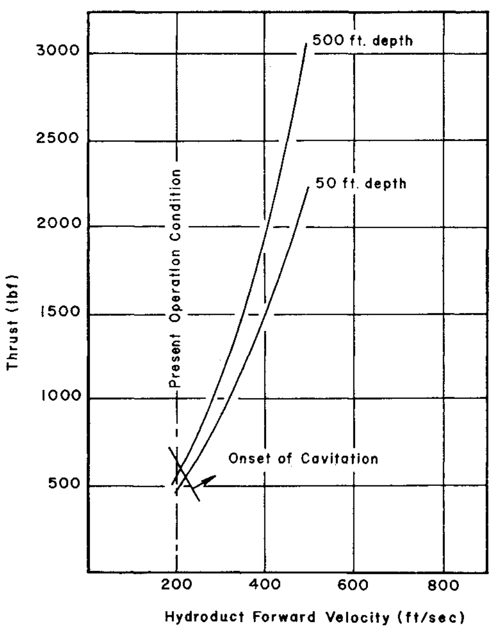

Fig. 6 Hydroduct thrust sensitivity to forward velocity and depth.

produces 496.3 lbf; a three-phase (solid-liquid-vapor) expansion produces a thrust of 435.42 lbf. Hence, it may be concluded that 400 lbf of drag will dominate the vehicle performance. An obvious alternative would be to use a higher specific impulse propellant. The heat release from a thermite type propellant is of the order of 3500 Btu/lbm, below satisfactory limits of our operational criteria. An improved propellant, with a minimum heat release of 4400 Btu/lbm, was used to evaluate a hydroduct design to insure an effective net thrust. The data and graphs are based upon this figure.

4) The cruising velocity of the hydroduct is determined by the launch velocity and is shown to be sensitive to all design parameters.

5) An efficient hydroduct can only be developed after a complete knowledge of all losses is available. Nozzle losses appear to dominate the operation and suggest further research into the nature of three-phase expansion processes.

Appendix A: Sample Thrust Calculation

The following calculations are presented for a hydroduct that operates at the highest attainable speed with minimum effect of cavitation. In shallow water operation, the body length-to-diameter ratio is critical to limit cavitation. Prescribed hydrodynamic shapes may reduce form drag losses attendant in a high-speed vehicle design. However, for even the best "bubble-free" drag profile that would also permit space adequate for cargo, propellant storage, combustion chamber, and nozzle, there would be about a minimum of 460 lb of drag induced by a hydroduct traveling at 200 fps and at a depth of 50 ft. For stable operation, such a device must be required to produce continuously at least 500 lb of total thrust.

A idealized cycle is considered below. Although calculations are made for several initial velocities, assuming constant drag losses, the net thrust available will vary for each design. If net thrust is positive for all design velocities, the hydroduct will perform in the desired manner. Total thrust may fall below the total drag value with a resulting decrease in forward velocity.

The following set of parameters is selected for the sample calculation: p_p = propellant density, 0.1 lbm/in.^{3}; A_p = area of propellant burning surface, 10 in.^{2}; ΔH_A = propellant heat release, 3500 Btu/lbm; m_w = water flow rate through hydroduct, 8 lbm/sec; r = propellant consumption rate, 1.6 lbm/sec; V_h = hydroduct forward velocity, 200 fps; d = depth of water above hydroduct, 50 ft; $\rho g/g_c$ = weight density of water, 64 lbf/ft^3; ρ = mass density of water, 64 lbm/ft^3. The foregoing parameters have been selected and represent a reasonably realistic design. In the following example, the assumption of 500 lb of thrust at V_h = 200 fps and d = 50 ft required the adjustment of the parameters through successive trail and error calculations. The following model is a result of these calculations.

A. Hydroduct chamber pressure: The pressure in the hydroduct chamber is equal to the sum of atmospheric, hydrostatic, and stagnation pressures.

$$P_c = P_A + \rho(g/g_c)d + \tfrac{1}{2}(\rho)\, V_h^2/g_c \tag{3}$$

$$P_c = 14.7 \text{ lbf/in.}^2 + \frac{64 \text{ lbf/ft}^3 \times 50 \text{ ft}}{144 \text{ in.}^2/\text{ft}^2} + \frac{64 \text{ lbm/ft}^3 \times (200 \text{ fps})^2}{2 \times 32.2 \text{ (lbm/lbf) (ft/sec}^2) \times 144 \text{ in./ft}}$$

$$P_c = 14.7 + 22.2 + 276 = 315 \text{ psia}$$

At a chamber pressure of 315 psia, it was assumed that negligible pressure drop occurs across the intake duct. The intake duct design is dependent only on the allowable pressure drop requirements. A minimum chamber pressure of 315 psia is critical to insure a uniform heat release from the propellant; thus, the intake pipe will have to be designed to minimize the pressure drop.

B. Propellant heat release: The propellant burning rate is given by $r = K_1 P_c^k$, so that for the design pressure of 315 psia, this propellant will be consumed at a rate r = 1.6 in./sec. A cigarette grain configuration of area A_p = 10 in.^{2} and heat of reaction ΔH_A = 3500 Btu/lbm, could be designed for the small space available. Therefore, the heat release rate of the thermite propellant

$$Q_A = (\rho_p)\,(rA_p)\,(\Delta H_A) \tag{4}$$

$$Q_A = \left(0.1 \frac{\text{lbm}}{\text{in.}^3}\right)\left(1.6 \frac{\text{in.}}{\text{sec}} \times 10 \text{ in.}^2\right)\left(3500 \frac{\text{Btu}}{\text{lbm}}\right)$$

$$Q_A = 5600 \text{ Btu/sec}$$

This heat is available to vaporize the intake water.

C. Expansion process: The intake water, at 60°F, at stagnation conditions is 315 psia. Hence, the specific enthalpy of the water in the combustion chamber, h_2 in BTU/ibm is

$$h_2 = Q_A/\dot{m}_w + V_h/2Jg_c + C_p\Delta T \tag{5}$$

$$h_2 = \frac{5600 \text{ Btu/sec}}{8 \text{ lbm/sec}} + \frac{(200 \text{ ft/sec})^2}{2 \times 788 \text{ ft-lbf/Btu} \times 32.2 \text{ ft-lbm/lbfsec}^2} + (1 \text{ Btu/lbm°F})\,(60° - 32°)$$

$$h_2 = 700 + 0.79 + 28 = 728.79 \text{ Btu/lbm}$$

The quality of the steam in the combustion chamber is

$$x = \frac{h_2 - h_{2f}}{h_{2fg}} = \frac{728.79 - 398.67}{804.5} = 0.41$$

C-1. Equilibrium expansion: If the mixture is assumed to be in equilibrium and the water droplets and steam vapor at the same velocity at any given transverse section, then the adiabatic expansion of wet steam through the nozzle can be determined. The 41% quality steam at 315 psia would have an entropy of

$$S_2 = S_{f_{315\text{psia}}} + xS_{fg_{315\text{psia}}} = 0.5934 + 0.41\,(0.912) = 0.967 \text{ Btu/lbm°F}$$

and the isentropic expansion requires that the 37 psia mixture of the nozzle exit be at an entropy of S_1 = 0.967 Btu/lbm°F. The quality of the mixture at the nozzle exit would be

$$x_{1_{37\text{psia}}} = \frac{S_1 - S_{1f}}{S_{1fg}} = \frac{0.9670 - 0.3854}{1.2972} = 0.4482$$

Hence, the specific enthalpy of the nozzle exit would be

$$h_1 = h_{f_{37\text{psia}}} + xh_{fg_{37\text{psia}}} = 231.26 + 0.4482\,(936.9) = 651.26 \text{ Btu/lbm}$$

The relative exhaust velocity would therefore be

$$V_f = 223.78\,(h_2 - h_1)^{1/2} = 223.78\,(728.7 - 651.26) = 1965 \text{ fps}$$

where $V_f^2 > V_h^2$. The thrust available for this water steam mixture expanded isentropically is then (See Fig. 3)

$$T = \frac{8 \text{ lbm/sec}}{32.2 \text{ (lbm/lbf) (ft/sec}^2)}\,(1965 - 200) \text{ fps} = 438.5 \text{ lbf}$$

C-2. Two-phase expansion: For expansion of the mixture of liquid and vapor through the nozzle in which the liquid droplets lag behind the steam, the velocities of the water and vapor are calculated from their respective saturated values.

For the *saturated vapor* component,

$$P_2 = 315 \text{ psia}, x_2 = 1.00, h_2 = 1203.25 \text{ Btu/lbm}$$
$$S_2 = 1.5061 \text{ Btu/lbm°F}$$

Isentropic expansion to 37 psia requires $S_1 = 1.5061$ Btu/-lbm°F. Hence, the quality of the vapor at the nozzle exit is

$$x_1 = \frac{S_1 - S_f}{S_{fg}} = \frac{1.5061 - 0.3854}{1.2972} = 0.8369$$

and the enthalpy of the vapor is, therefore,

$$h_1 = 231.26 + 0.8639\ (936.9) = 1040.65 \text{ Btu/lbm}$$

Finally, the saturated vapor would accelerate to a velocity of

$$V_g = 223.78\ (h_2 - h_1)^{1/2} = 223.78\ (1203.25 - 1040.65)^{1/2} = 2853 \text{ fps}$$

where $V_g^2 \gg V_h^2$.

For the *saturated liquid* component,

$$P_2 = 215 \text{ psia}, x_2 = 0.00, h_2 = 398.7 \text{ Btu/lbm}$$
$$S_2 = 0.5934 \text{ Btu/lbm°F}, P_1 = 37 \text{ psia}$$
$$x_1 = (0.5934 - 0.3854)/1.2972 = 0.1603$$
$$S_1 = 0.5934 \text{ Btu/lbm°F}$$
$$h_1 = 231.26 + 0.1603\ (936.9) = 381.44 \text{ Btu/lbm}$$

Finally, the saturated liquid will accelerate to a velocity of

$$V_f = 223.78\ (h_2 - h_1)^{1/2} = 223.78 \times (398.7 - 381.44)^{1/2} = 993 \text{ fps}$$

where $V_f^2 \gg V_h^2$. For comparison with C-1, the thrust available from the water steam mixture expanded isentropically as separate components would be

$$T = m_g/g_c(V_g - V_h) + m_f/g_c(V_c - V_h)$$

$$T = \frac{0.41 \times 8 \text{ lbm/sec}}{32.2 \text{ (lbm/lbf/fps}} (2853 - 200) \text{ fps} + \frac{0.59 \times 8 \text{ lbm/sec}}{32.2 \text{ (lbm/lbf) fps}} (933 - 200) \text{ fps}$$

$$T = 270.24 + 107.44,\ T = 377.64 \text{ lbf}$$

The equilibrium thrust was calculated to be 438.5 lbf whereas the two-phase thrust was calculated to be 377.64 lbf, so that the actual thrust will be somewhat lower because of the losses that were neglected in the calculations.

C-3. Three-phase expansion: The solid reaction products can also contribute to the thrust. The exact physical mechanism whereby the small solid particles transfer their heat content to the surrounding fluid as they are accelerated to the exhaust velocity is not well known. However, the thrust contribution can be approximated by assuming an effective velocity of the solid particles through the relation

$$V_{s.p} = 223.78\ (\Delta h)^{1/2}$$

where the change in enthalpy is derived from the reaction products whose specific heat is approximately 0.17 Btu/-lbm°F, and whose temperature change from the combustion chamber (421°F) to the nozzle exit (262°F) caused the solid particles to release 27 Btu/lbm,

$$V_{s.p} = 223.78\ (27)^{1/2} = 1164 \text{ fps}$$

Therefore, the thrust contribution from the solid particles would be

$$T = \frac{m_p}{g_c} V_{s.p} = \frac{1.6 \text{ lbm/sec}}{32.2 \text{ (lbm/lbf) fps}} \times 1164 \text{ fps}$$

$$T = 57.8 \text{ lbf}$$

The total theoretical thrust would be $T = 438.5 + 57.8 = 496.3$ lbf (equilibrium and solid particles); $T = 337.64 + 57.8 = 435.45$ lbf (isentropic and solid particles). The thrust will be smaller than computed by the aforementioned idealized processes by considering frictional losses. In addition to expansion losses, there are losses introduced by the intake pipe and nozzle configuration. These effects could cause excessive pressure drops so that the combustion chamber pressure is incapable of maintaining the required burning rate. With the small positive net thrust available and the exact magnitude of the losses, it is difficult to estimate the performance of hydroducts on a purely theoretical basis. In general, it is clear that over-all thrust and drag requirements suggest only marginal performance at best.

D. Efficiency: The over-all thermal efficiency of the hydroduct propulsion system can be calculated to be

$$\eta = \frac{m_p V_{s.p}^2 + w_g(V_g^2 - V_h^2) + m_f(V_f^2 - V_h^2)}{2J_c Q_A} \times 100\%$$

$$\eta = \frac{(1.6 \text{ lbm/sec})\ (1164 \text{ fps})^2 + (0.41 \times 8 \text{ lbm/sec})\ (2853^2 - 200^2) \text{ ft}^2/\text{sec}^2}{2 \times 778 \text{ ft-lbf/Btu} \times 5600 \text{ Btu/sec}} + \frac{0.59 \times 8\ (933^2 - 200^2)}{2 \times 778 \times 5600} \times 100\%$$

$$\eta = \frac{2{,}167{,}834 + 26{,}566{,}717 + 2{,}200{,}708}{280{,}577{,}920} \times 100\%$$

$$\eta = \frac{20{,}955{,}259}{280{,}577{,}920} \times 100\% = 11.03\%$$

Note that although the vaporized steam contributes about 85% of the over-all thermal efficiency of the hydroduct, it contributes only about 62% of the thrust.

References

[1] Brady, J. F., *Astronautics and Aeronautics*, Vol. 3, March 1965, pp. 50–54.

[2] Hacker, D. S. and Lieberman, P., *Society of Automotive Engineers Journal*, Oct. 1961, pp. 74–75.

[3] Hacker, D. S. and Lieberman, P., "Evaluation of an Underwater hydroduct and Hydroductor," Rept. D 143, Sept. 1958, Armour Research Foundation.

[4] Pai, S. I., *Fluid Dynamics of Jets*, Van Nostrand, Princeton, N. J., 1954, pp. 98–152.

[5] Yellot, S., *Transactions of the American Society of Mechanical Engineers*, Vol. 59, 1937.

[6] Kennan, J. H., *Thermodynamics*, Wiley, New York, 1941, pp. 442–446.

[7] Eddington, R. B., *Investigation of Shock Phenomena in a Supersonic Two-Phase Tunnel*, Paper 66-87, 1966, AIAA.

[8] Church, E. F., *Steam Turbines*, McGraw-Hill, New York, 1935, pp. 96–99.